Appendix 1: Derivation

In this appendix we derive the CES demand function with willingness to pay.

$$x_i = u + - \sigma.(p_i - \sum_{i=1}^{n} S_i.p_i) + \sigma(b_i - \sum_{i=1}^{n} S_i.b_i) \qquad (1)$$

Maximize CES utility function:

$$U = B^{-\frac{1}{\rho}}.X. = B^{-\frac{1}{\rho}}.\left[\sum_{i=1}^{n} \frac{B_i}{B}.(X_i)^{-\rho}\right]^{-\frac{1}{\rho}} \qquad (2)$$

Where: i is the source country (where there are n countries, $i \in 1...n$);

U is utility derived from total consumption;

X is the composite quantity of all goods;

B is the composite distribution parameter;

B_i are the distribution parameters of goods from country I *(willingness to pay variable)*;

X_i is the quantity of goods from country i; and

ρ is a substitution parameter. It is related to the elasticity of substitution between goods from different countries i, $(\sigma = \frac{1}{1+\rho})$.

Subject to budget constraint:

$$Y = [\sum_{i=1}^{n} P_i.X_i] \qquad (3)$$

where: P_i is the price of the good from country i;

Y is income or value of goods; and

Gives:

$$\Lambda. P_i = -\frac{1}{\rho}.B^{-\frac{1}{\rho}}.\left[\sum_{i=1}^{n}\frac{B_i}{B}.(X_i)^{-\rho}\right]^{-\frac{1}{\rho}-1}.-\rho\frac{B_i}{B}.(X_i)^{-\rho-1}$$

$$P_i = \Lambda^{-1}.B^{-\frac{1}{\rho}}.X^{(1+\rho)}.\frac{B_i}{B}.(X_i)^{-(\rho+1)} \qquad (4)$$

Where: Λ^{-1} is the shadow price.

Rearrange 4:

$$X_i = P_i^{-\sigma}\Lambda^{-\sigma}B^{-\frac{1}{\rho(1+\rho)}}.X.\left(\frac{B_i}{B}\right)^{\sigma} \qquad (5)$$

Where: $\sigma = 1/(1+\rho)$

Note:

$$\frac{X_i}{X_j} = \frac{(P_i/B_i)^{-\sigma}}{(P_j/B_j)^{-\sigma}}$$

The ratio of the quantities depends on the ratio of prices adjusted for willingness to pay.

Hence it is willingness-to-pay-adjusted prices that matter, not just prices.

In proportionate changes (5) becomes:

$$x_i = -\sigma.p_i + -\sigma.\lambda - \frac{1}{\rho(1+\rho)}b + x + \sigma(b_i - b) \qquad (6)$$

Where: lowercase represents the proportionate change in the equivalent uppercase variable.

Equation (6) is very close to equation (1). The only difference is λ. To define λ we must look at the utility function (2) and convert to proportionate changes:

$$x = -\frac{1}{\rho}.\sum_{i=1}^{n}S_i.(b_i - b + -\rho x_i)$$

Where: $S_i = \dfrac{\frac{B_i}{B}.(X_i)^{-\rho}}{\sum_{i=1}^{n}\frac{B_i}{B}.(X_i)^{-\rho}}.$

If we define b as $\sum_{i=1}^{n} S_i \cdot b_i$, then:

$$x = \sum_{i=1}^{n} S_i \cdot x_i \qquad\qquad (7)$$

Substitute (6) into (7) and rearrange to get an equation for λ:

$$x = \sum_{i=1}^{n} S_i \cdot \left(-\sigma \cdot p_i + -\sigma \cdot \lambda - \frac{1}{\rho(1+\rho)} b + x + \sigma(b_i - b) \right)$$

$$\sigma\lambda = -\sigma \sum_{i=1}^{n} S_i \cdot p_i - \frac{1}{\rho(1+\rho)} b \qquad\qquad (8)$$

Next substitute (8) into (6) to get our demand equation for x_i:

$$x_i = -\sigma \cdot p_i + \sigma \sum_{i=1}^{n} S_i \cdot p_i + \frac{1}{\rho(1+\rho)} b - \frac{1}{\rho(1+\rho)} b + x + \sigma(b_i - b)$$

$$x_i = x + -\sigma \cdot \left(p_i - \sum_{i=1}^{n} S_i \cdot p_i \right) + \sigma\left(b_i - \sum_{i=1}^{n} S_i \cdot b_i \right)$$

Finally, note that the value shares can be used for determining the shares,

$$S_i = \frac{\frac{B_i}{B}\cdot(X_i)^{-\rho}}{\sum_{i=1}^{n}\frac{B_i}{B}\cdot(X_i)^{-\rho}} = \frac{P_i \cdot X_i}{\sum_{i=1}^{n} P_i \cdot X_i}.$$

The equivalence of the two shares can be shown by substituting (2) into (4) and rearranging:

$$P_i = \Lambda^{-1}\cdot B^{-\frac{1}{\rho}}\cdot X^{(1+\rho)}\cdot\frac{B_i}{B}\cdot(X_i)^{-(\rho+1)}$$

$$P_i = \Lambda^{-1}U\cdot\left(\sum_{i=1}^{n}\frac{B_i}{B}\cdot(X_i)^{-\rho}\right)^{-1}\cdot\frac{B_i}{B}\cdot X_i^{-\rho}\cdot X_i^{-1}$$

$$\frac{P_i \cdot X_i}{\Lambda^{-1}\cdot U} = \frac{\frac{B_i}{B}\cdot X_i^{-\rho}}{\sum_{i=1}^{n}\frac{B_i}{B}\cdot X_i^{-\rho}}$$

Appendix 2: Implementation in a CGE model

We incorporate the willingness to pay theory outlined above into a CGE model — the ImpactECON global supply chain model (Walmsley and Minor, 2016a and 2016b). The supply chain model and database is an updated version of that developed in Walmsley, Hertel and Hummels (2014) and Hertel, Hummels and Walmsley (2014).[1] This appendix should be read in conjunction with the model code; a familiarity with GTAP model notation is assumed and users are referred to the code for definitions of the variables. The RunGTAP application, including code are available at: https://impactecon.com/resources/cge-models/.

Like most GEMPACK-based (Harrison, Horridge, Jerie and Pearson, 2014) models, this model is written in percent changes. Those implementing this in the levels (e.g., in GAMS), should note that the distribution parameters should be included in addition to the other parameters used to calibrate the equation. The new distribution parameter (B_i) should be set equal to the price, and the existing (fixed) parameter (A_i)[2] should continue to be used to calibrate the equation. This reflects that fact that empirical estimates of changes in WTP are likely to be determined with the initial price of the commodity in mind. The composite distribution parameter can be calculated in the same way as the composite price.

$$U = B^{-\frac{1}{\rho}}.X. = B^{-\frac{1}{\rho}}.\left[\sum_{i=1}^n A_i \frac{B_i}{B}.(X_i)^{-\rho}\right]^{-\frac{1}{\rho}}$$

---

[1] The supply chain database has been further revised with updated BEC concordances produced by the UN. Minor changes were also made to separate investment goods from final demand and to the balancing programs and model code.

[2] Since this parameter is fixed (zero change) it is not needed in the percent change implementation.

Below is a detailed description of the changes made to the ImpactECON supply chain model. The ImpactECON supply chain model differs from the GTAP model in that it tracks imports by both agent and source. The model relies on a detailed database of supply chains that contains the value of imports of commodity i, from region s, purchased by firm j located in region r, as well as bilateral imports of commodity i purchased by households, and by the government. With this additional data the model can account for how the TFA will impact OECD households' imports of light manufacturing from middle income countries differently from firms' demand for the same products. In Figure 2 the structure of the supply chain model is compared to the GTAP model structure. The figure illustrates three separate Arminigton equations in the supply chain model for each agent (*qips-private households*, *qifs-firms*, and *qigs-government*)[3], as compared to the GTAP model where all agent imports are aggregated (*qxs*). This allows us to apply the iceberg and willingness-to-pay parameters directly into each agent's Armington equation (Equations (1A), (2A) and (3A)), facilitating decomposition and comparison of the results by agent.[4]

Private household (1A):

$$qips(i,r,s) = qpm(i,s) - aips(i,r,s)$$
$$-ESUBM(i).\big(ppms(i,r,s) - aips(i,r,s) - ppm(i,s)\big)$$
$$+ESUBM(i).\big(dppms(i,r,s) - dppim(i,s)\big)$$

$$dppim(i,s) = \text{sum}(k, \text{REG}, \text{PMSHRS}(i,k,s) \ * [dppms(i,k,s)]$$

Government (2A):

$$qigs(i,r,s) = qgm(i,s) - aigs(i,r,s)$$
$$-ESUBM(i).\big(pgms(i,r,s) - aigs(i,r,s) - pgm(i,s)\big)$$
$$+ESUBM(i).\big(dpgms(i,r,s) - dpgim(i,s)\big)$$

$$dpgim(i,s) = \text{sum}(k,REG,GMSHRS(i,k,s) * [dpgms(i,k,s)]$$

Firms (3A):

$$qifs(i,j,r,s) = qfm(i,j,s) - aifs(i,j,r,s)$$
$$- ESUBM(i).\big(pfms(i,j,r,s) - aifs(i,j,r,s) - pfm(i,j,s)\big)$$
$$+ ESUBM(i).\big(dpfms(i,j,r,s) - dpfim(i,j,s)\big)$$

$$dpfim(i,j,s) = \text{sum}(k,REG,FMSHRS(i,j,k,s) * [dpfms(i,j,k,s)]$$

Where: *i* is the commodity, j is the sector, *r* is the source, *s* is the destination;

ESUBM is the elasticity of substitution between imports from different sources;

PMSHRS, GMSHRS and *FMSHRS* are the relevant value shares;

aifs, *aips* and *aigs* represent the iceberg variable (*ams* in the GTAP model) for firms, private and government agents respectively;

dpfms, *dppms* and *dpgms* represent the distribution parameters on imports by commodity, agent, source and destination;

dpfim, *dppim* and *dpgim* the composite distribution parameters by commodity for firms, private and government agents for the willingness-to-pay method;

pfms, *ppms* and *pgms* represent the prices by commodity, agent, source and destination; and

pfm, *ppm* and *pgm* represent the composite prices by commodity, agent, and destination.

If the willingness to pay shocks are compensated, then the composite distribution parameters *dpfim*, *dppim* and *dpgim* are all zero and nothing more is required. If, however, the willingness to pay is uncompensated, the willingness to pay shocks filter through to the other levels of the demand structure. For instance, at the second level of the supply chain model:

Private household (4A):

$$qpm(i,s) = qp(i,s) - ESUBD(i).\big(ppm(i,s) - pp(i,s)\big)$$
$$+ESUBD(i).\big(dppim(i,s) - dpp(i,s)\big)$$

$$qpd(i,s) = qp(i,s) - ESUBD(i).\big(ppd(i,s) - pp(i,s)\big)$$
$$+ESUBD(i).\big(dps(i,s) - dpp(i,s)\big)$$

$$dpp(i,s) = PMSHR(i,s) * dppim(i,s) + [1 - PMSHR(i,s)] * dps(i,s)$$

Government (5A):

$$qgm(i,s) = qg(i,s) - ESUBD(i).\big(pgm(i,s) - pg(i,s)\big)$$
$$+ESUBD(i).\big(dpgim(i,s) - dpg(i,s)\big)$$

$$qgd(i,s) = qg(i,s) - ESUBD(i).\big(pgd(i,s) - pg(i,s)\big)$$
$$+ESUBD(i).\big(dps(i,s) - dpg(i,s)\big)$$

$$dpg(i,s) = GMSHR(i,s) * dpgim(i,s) + [1 - GMSHR(i,s)] * dps(i,s)$$

Firms (6A):

$$qfm(i,j,s) = qf(i,j,s) - ESUBD(i).\big(pfm(i,j,s) - pf(i,j,s)\big)$$
$$+ESUBD(i).\big(dpfim(i,j,s) - dpf(i,j,s)\big)$$

$$qfd(i,j,s) = qf(i,j,s) - ESUBD(i).\left(pfd(i,j,s) - pf(i,j,s)\right)$$
$$+ESUBD(i).\left(dps(i,s) - dpf(i,j,s)\right)$$

$$dpf(i,j,s) = FMSHR(i,j,s) * dpfim(i,j,s) + [1 - FMSHR(i,j,s)]$$
$$* dps(i,s)$$

Where: *ESUBD* is the elasticity of substitution between domestic and imported goods;

PMSHR, GMSHR and *FMSHR* are the relevant value shares;

qfm, *qpm* and *qgm* represent the quantity of imports of commodity i by firms, private and government agents respectively;

qfd, *qpd* and *qgd* represent the quantity of domestically sourced commodity by firms, private and government agents respectively;

qf, *qp* and *qg* represent the quantity of commodity i demanded by firms, private and government agents respectively;

pfm, *ppm* and *pgm* represent the prices of imports of commodity i paid by firms, private and government agents respectively;

pfd, *ppd* and *pgd* represent the prices of domestically sourced commodity i by firms, private and government agents respectively;

pf, *pp* and *pg* represent the composite price of commodity i paid by firms, private and government agents respectively; and

dpf, dpp and *dpg* the composite distribution parameters firms, private and government agents for the willingness-to-pay method.

It is assumed in the above equations that domestically produced goods are also affected by changes in the willingness to pay ($dps(i,s)$). For instance, to the extent that a good is

produced using imported intermediate inputs in the production process. This mechanism can be turned off by the user.

Production (7A):

$$dps(j, s) = \text{sum}(i, \text{TRAD}_{\text{COMM}}, \text{STC}(i, j, r) * [dpf(i, j, s)])$$

Where: *STC* are the cost shares for production of commodity i, by sector j;

Finally, at the top level:

Private household (8A):

$$
\begin{aligned}
qp(i, s) - pop(s) \\
&= \text{sum}(k, \text{TRAD\_COMM}, \text{EP}(i, k, s) * pp(k, s)) + \text{EY}(i, s) \\
&* [yp(s) - pop(s)] \\
&- \text{sum}(k, \text{TRAD\_COMM}, \text{EP}(i, k, s) * [dpp(k, s) - dppall(s)])
\end{aligned}
$$

$$dpall(s) = \text{sum}(i, \text{TRAD\_COMM}, \text{VPA}(i, s) / \text{PRIVEXP}(s) * (dpp(i, s)))$$

Government (9A):

$$
\begin{aligned}
qg(i, s) &= qgov(i, s) - (pg(i, s) - pgov(i, s)) \\
&+ (dpg(i, s) - dpgall(i, s))
\end{aligned}
$$

$$dgall(s) = \text{sum}(i, \text{TRAD\_COMM}, \text{VGA}(i, s) / \text{GOVEXP}(s) * (dpg(i, s)))$$

Where: *pop* represent the quantity of imports of commodity i by firms, private and government agents respectively;

EP and *EY* are the price and income elasticities;

yp is price expenditure;

qgov is the quantity of demanded by government;

VPA, *VGA*, *PRIVEXP* and *GOVEXP* are the values for calculating the relevant shares; and

dppall and *dpgall* the composite distribution parameters private and government agents for the willingness-to-pay method.

Not that firms are not impacted at the top level as a Leontief production function is assumed.

These changes in willingness to pay affect all agents'—firms, households and firms—demand for imports, including the decision of whether to save or consume. Since the 'savings' good is dependent on the capital or investment good assembled in each region, demand for savings may also be affected by the change in preferences. To the extent that capital goods consist of the 'more preferable' imported commodities, households may choose to save more as a result of the reduction in time to trade.[5] Since savers will change their savings behavior as a result of the reduction in time to trade, it is reasonable that investment, funded by that savings, will also be affected by the reduction in time to trade. To include this in the model we assume that the increase in willingness to save flows through to the investment allocation, such that countries with greater willingness to pay for capital goods are also allocated more of the global savings (i.e., more investment occurs in that country). The total distribution parameter for savings is given by:

Savings (10A):

$$dpsall(s) = sum(c, CGDS\_COMM, dps(c,s))$$
$$+ sum(r, REG, [[NETINV(r) - SAVE(r)] / GLOBINV]$$
$$* sum(c, CGDS\_COMM, dps(c,r)));$$

---

[5] Again, this mechanism can be turned off by the user.

Where: c is the capital goods sector or the sector that compiles the investment good;

NETINV is net investment;

SAVE is the value of savings; and

GLOBINV is global net savings.

The impact of the changes in the distribution parameters are then incorporated into the equivalent variation measure and into the Hertel and Huff (2001) welfare decomposition. The impact on EV of the changes in the distribution parameters ($CNTwtp$) is a value weighted sum of the underlying changes in the distribution parameters (*dppall, dpgall* and *dpsall*), where the weights depend on the value shares of private consumption, government consumption and savings.[6] This is most easily achieved by summing the changes in the composite distribution parameters at the top level, since the composite distribution parameters at the top level are aggregates of the lower level distribution parameters that include changes in the willingness to pay of both domestic and imported goods purchased by all agents. The willingness to pay contribution to EV in the welfare decomposition is therefore given by (11A):

$$CNTwtp(s) = [0.01 * \text{EVSCALFACT(s)}]*$$
$$[\text{UELASPRIV(s)} * \text{PRIVEXP(s)} * dppall(s)$$
$$+ \text{GOVEXP(s)} * dpgall(s)$$
$$+ \text{SAVE(s)} * dpsall(s)]$$

---

[6] UELASPRIV is the elasticity of cost with respect to the utility from private consumption. This adjustment on the weight of private consumption stems from non-homothetic preferences. EVSCALFACT similarly scales welfare to take account of non-homothetic preferences in private consumption.

Figure 2: Production and Armington structures

A: GTAP and MyGTAP	B: ImpactECON Supply Chain Model
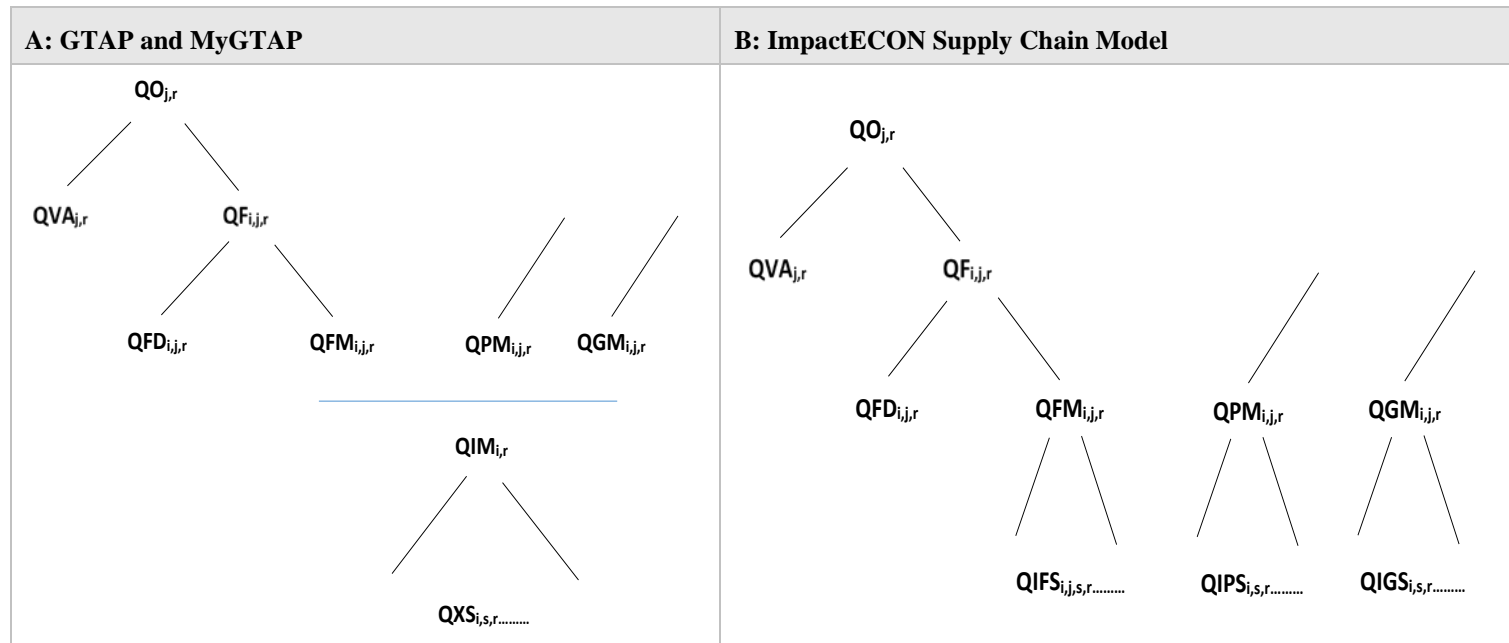
Source:

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